Static and Kinetic Friction in Frenkel-Kontorova-like Models

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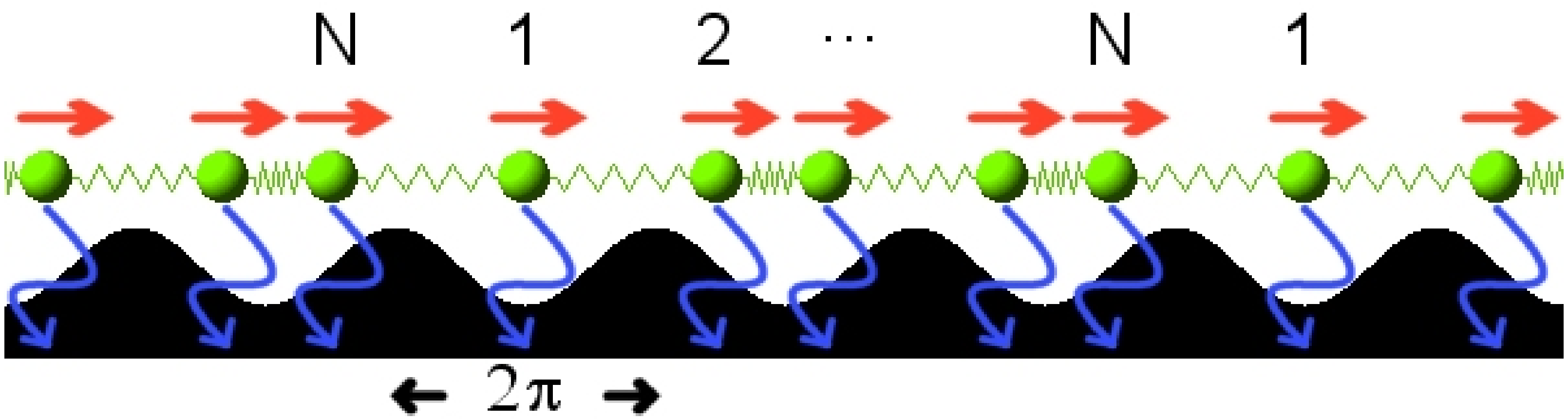
Overview

1. The Frenkel-Kontorova (FK) Model
2. The Frenkel-Kontorova-Tomlinson (FKT) Model
3. Static friction
4. Kinetic friction
5. Pinning $\longleftrightarrow$ Sliding
6. Why studying toy models?

1. The Frenkel-Kontorova (FK) Model



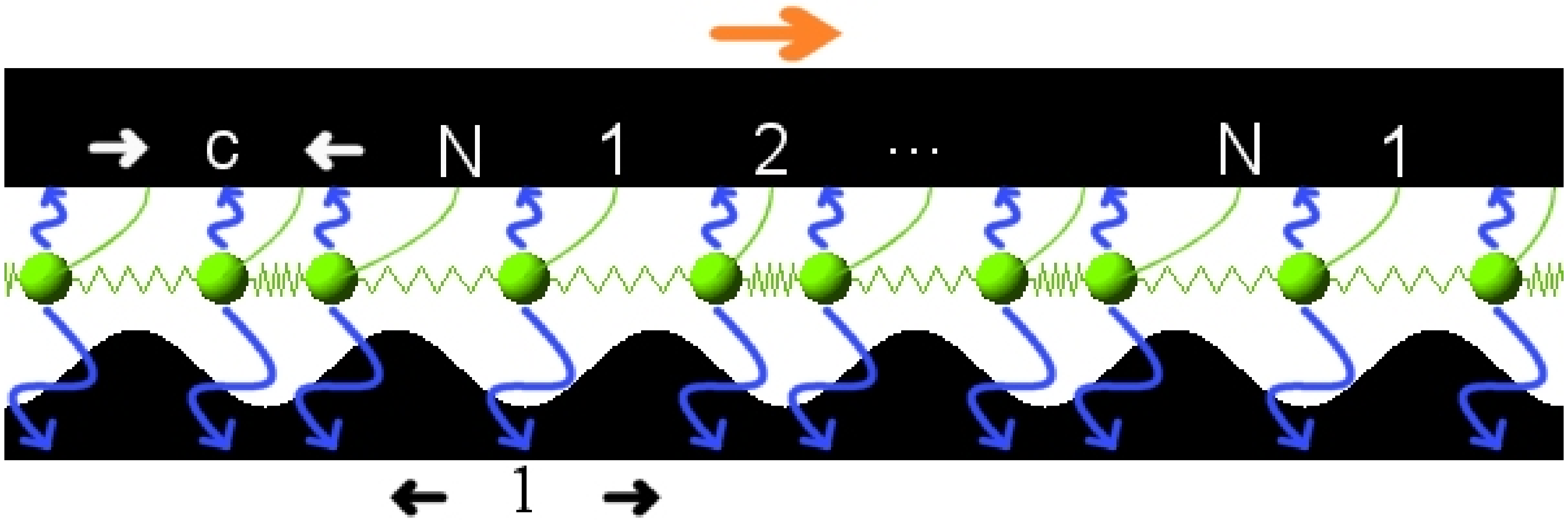
Equation of motion:

$$\ddot{x}_j + \gamma \dot{x}_j = x_{j-1} + x_{j+1} - 2x_j - b \sin x_j + F, \quad j = 1, \ldots, N$$

Boundary conditions:

$$x_{j+N} = x_j + 2\pi M, \quad \text{averaged distance:} \quad a = 2\pi \frac{M}{N}$$

2. The Frenkel-Kontorova-Tomlinson (FKT) Model



Equation of motion:

$$\ddot{\xi}_j + \underbrace{(\gamma_L + \gamma_U)}_{\gamma} \dot{\xi}_j = \xi_{j-1} + \xi_{j+1} - (2+\kappa)\xi_j + b \sin 2\pi \underbrace{(\overbrace{vt}^{x_B} + cj + \xi_j)}_{x_j}, \quad j = 1, \ldots, N$$

Boundary conditions:

$$\xi_{j+N} = \xi_j, \quad \Rightarrow \quad c = \frac{M}{N}$$

Co-workers: Torsten Strunz, Michael Weiss

Publications FK model:

1. T. Strunz and F.J. Elmer, Phys. Rev. E **58**, 1601 (1998).

2. T. Strunz and F.J. Elmer, Phys. Rev. E **58**, 1612 (1998).

Publications FKT model:

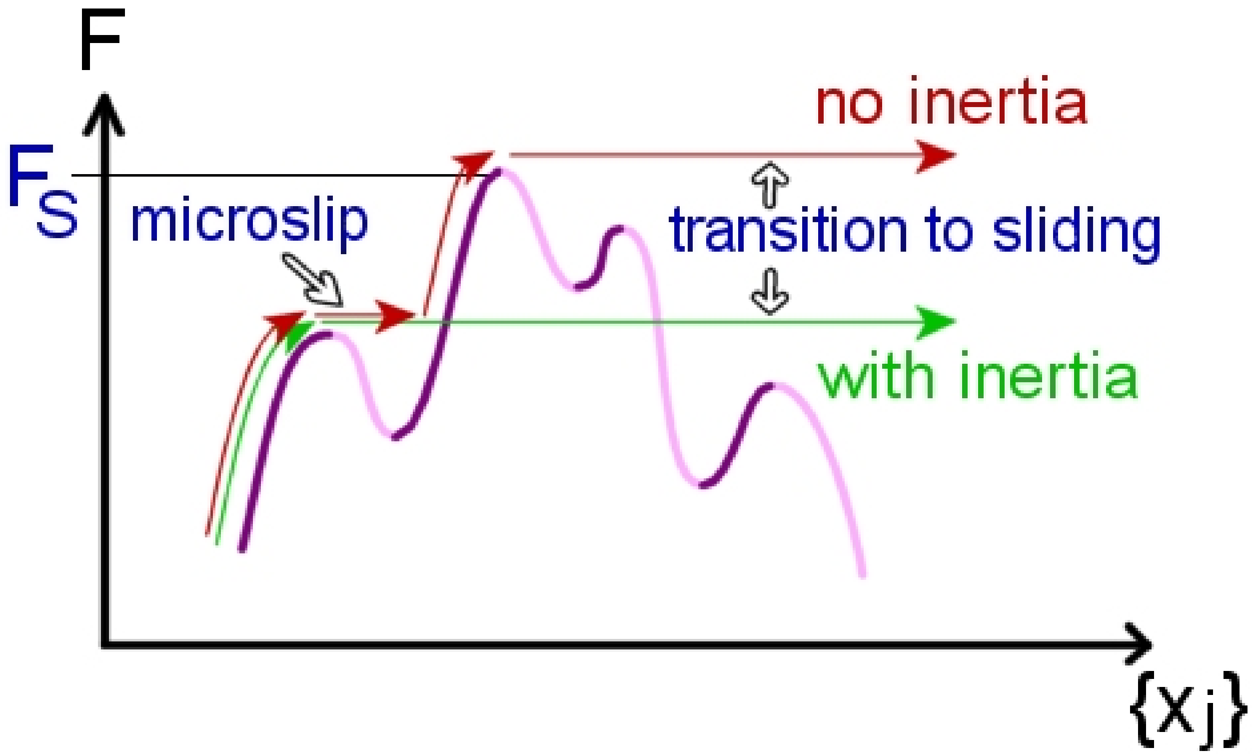
1. M. Weiss and F.J. Elmer, Phys. Rev. B. **53**, 7539 (1996).

2. M. Weiss and F.J. Elmer, Z. Phys. B. **104**, 55 (1997).

3. F.J. Elmer, in *Workshop on Friction, Arching, Contact Dynamics* edited by D. E. Wolf and P. Grassberger (World Scientific, Singapore, 1997).

3. Static friction

Phenomenological definition: *The force necessary to start sliding.*
⇒ Static friction depends on **history**, **inertia**, and **temperature**.



Unique definition: *Static friction F_S is the force where the last (meta)stable state disappears.*

Properties of the static friction in FK and FKT model

1. Ground state at $F = 0 \rightarrow$ last state at $F = F_S$
2. Commensurate lattic constants:

$$F_S \propto b^Q$$

   where $Q = N/\text{gcd}(M, N)$.

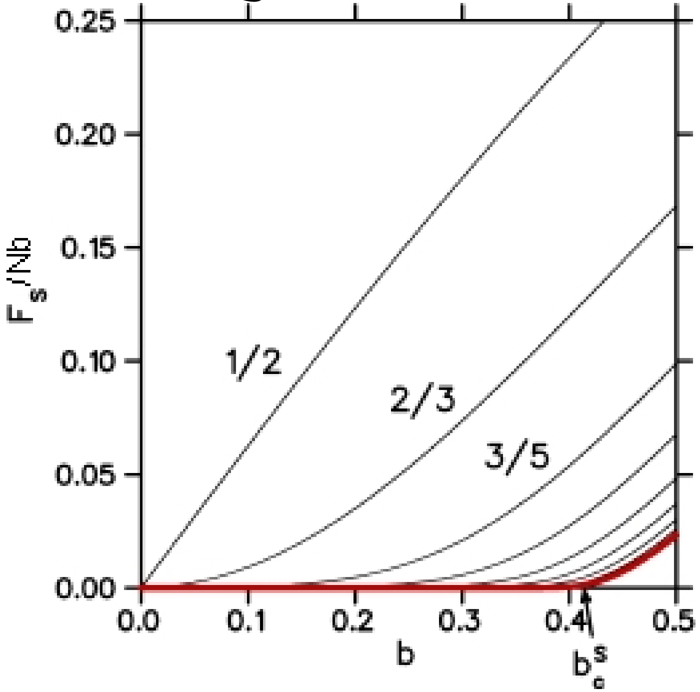
3. Incommensurate lattic constants:

$$F_S = \begin{cases} 0 & : b < b_c^S \\ const \cdot (b - b_c^S)^\alpha & : b > b_c^S \end{cases}$$

   with $\alpha = 3$ for FK and $\alpha = 2$ for FKT.

FKT model, $\kappa = 1$

$c \rightarrow$ golden mean



4. Kinetic friction

Definition: *The averaged force F_K necessary to slide with an on average constant velocity v.*

Dissipated energy per time unit: $F_K \cdot v$

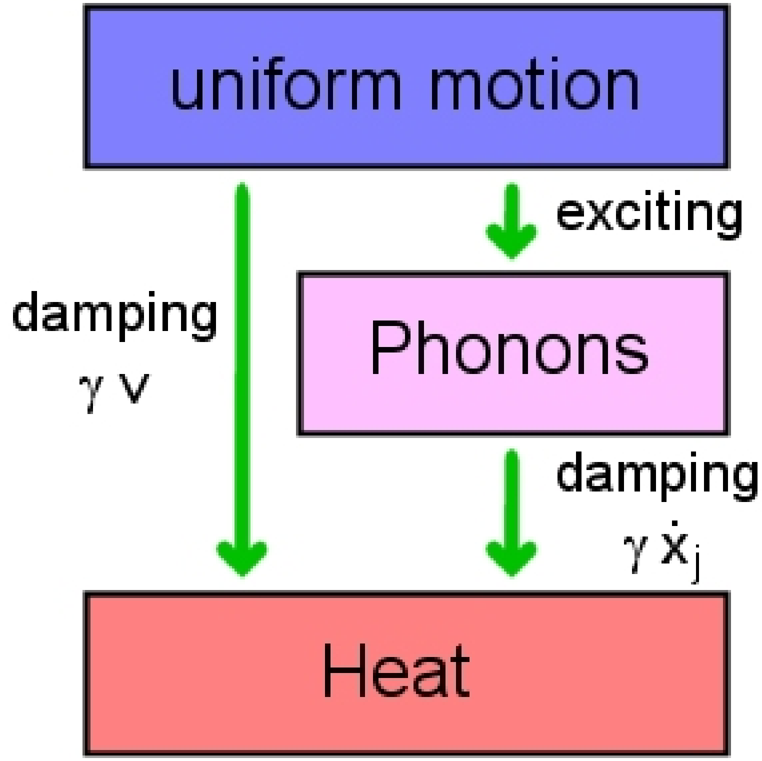
Averaged kinetic friction:

$$F = \frac{F_K}{N} = \left(\gamma_L + \gamma \left\langle\left\langle\left(\frac{\dot{x}_j}{v} - 1\right)^2\right\rangle_j\right\rangle_t\right) \cdot v$$

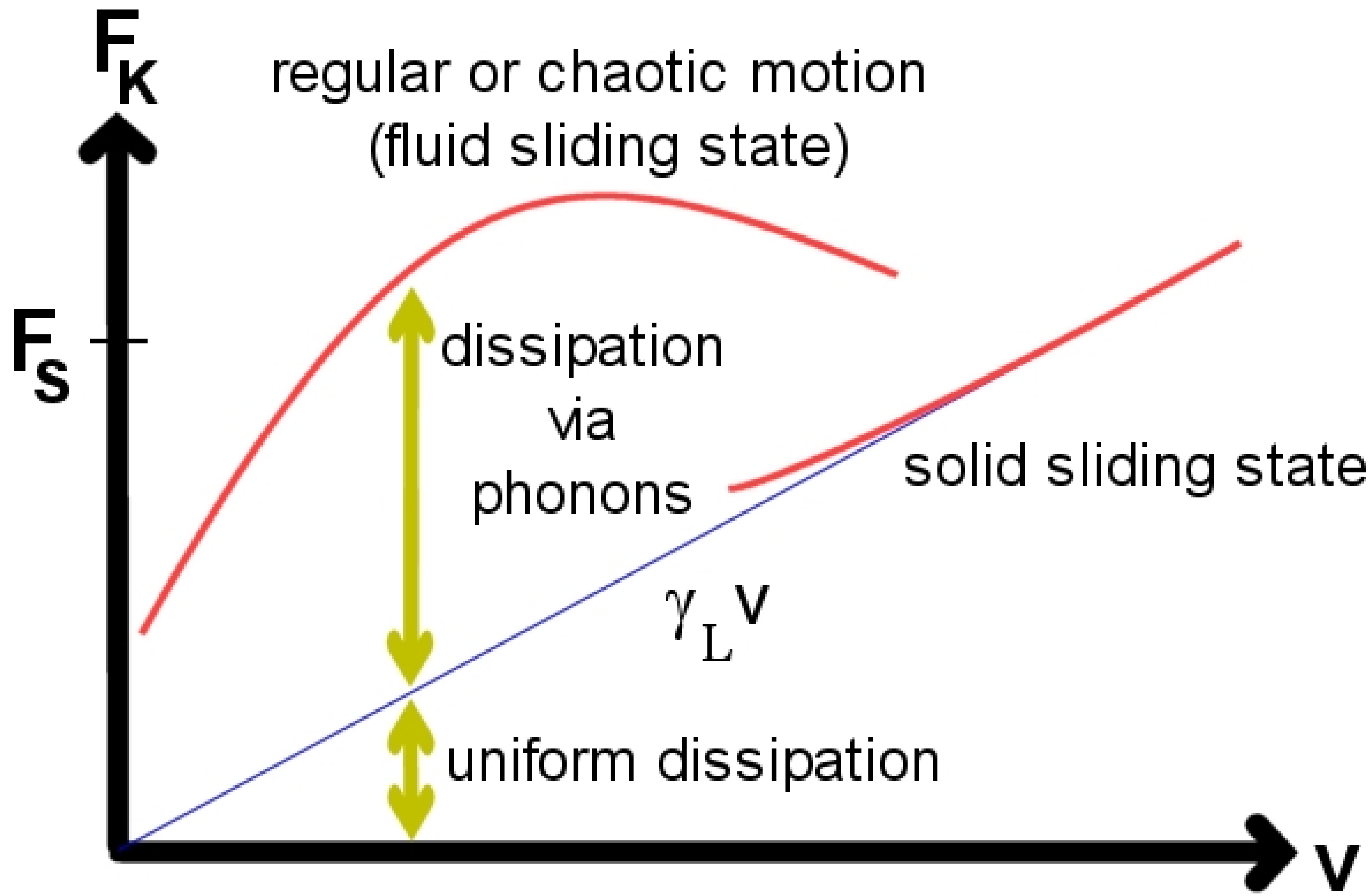
with the averaged velocity

$$v = \left\langle\left\langle \dot{x}_j \right\rangle_j\right\rangle_t$$

Dissipation channels:

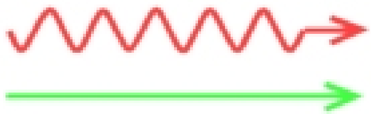


Kinetic friction versus velocity:



How uniform motion excites phonons?

The periodic substrate acts like a wave:



washboard wave: wavenumber K, frequency $\Omega \sim v$

phonon: wavenumber k, frequency $\omega(k)$

Dispersion relation:

$$\text{FK model:} \quad \omega(k) = 2|\sin(k/2)|$$

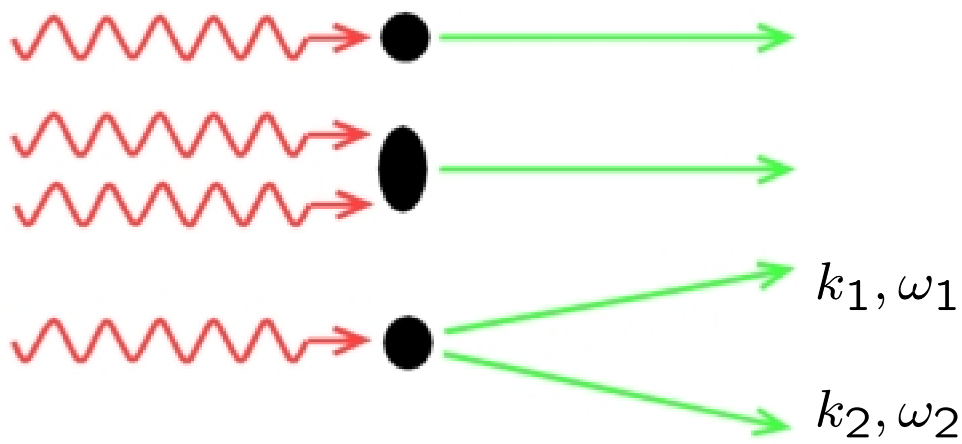
$$\text{FKT model:} \quad \omega(k) = \sqrt{\kappa + 4\sin^2(k/2)}$$

Resonances:

Main resonance:

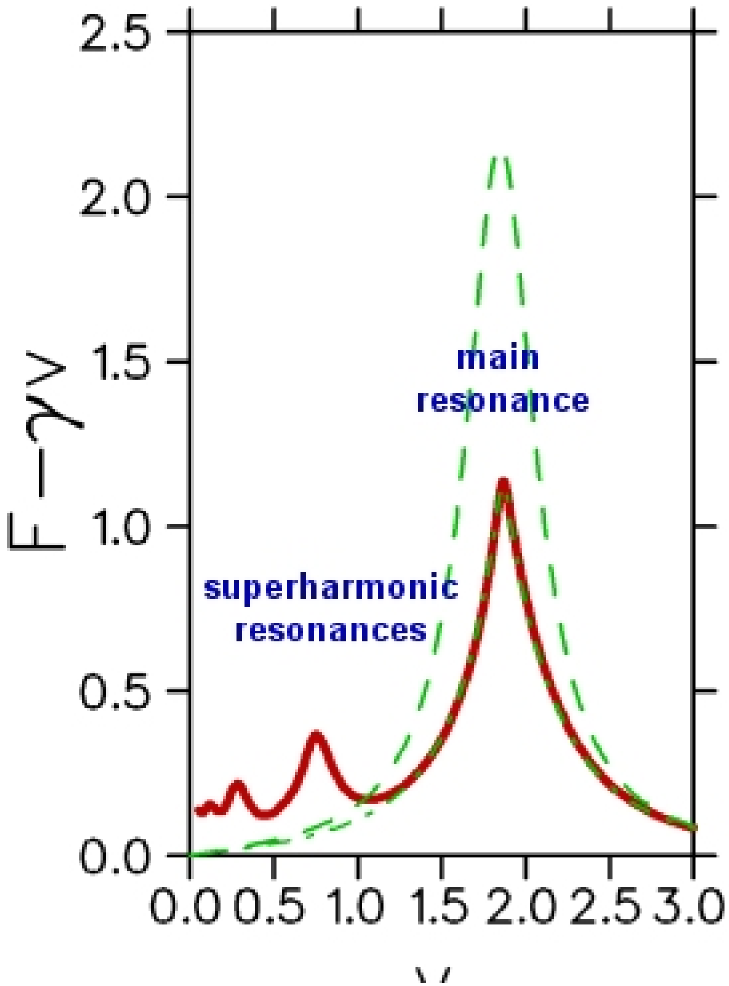
Superharmonic resonance of 2. order:

Parametric resonance of 1. order:

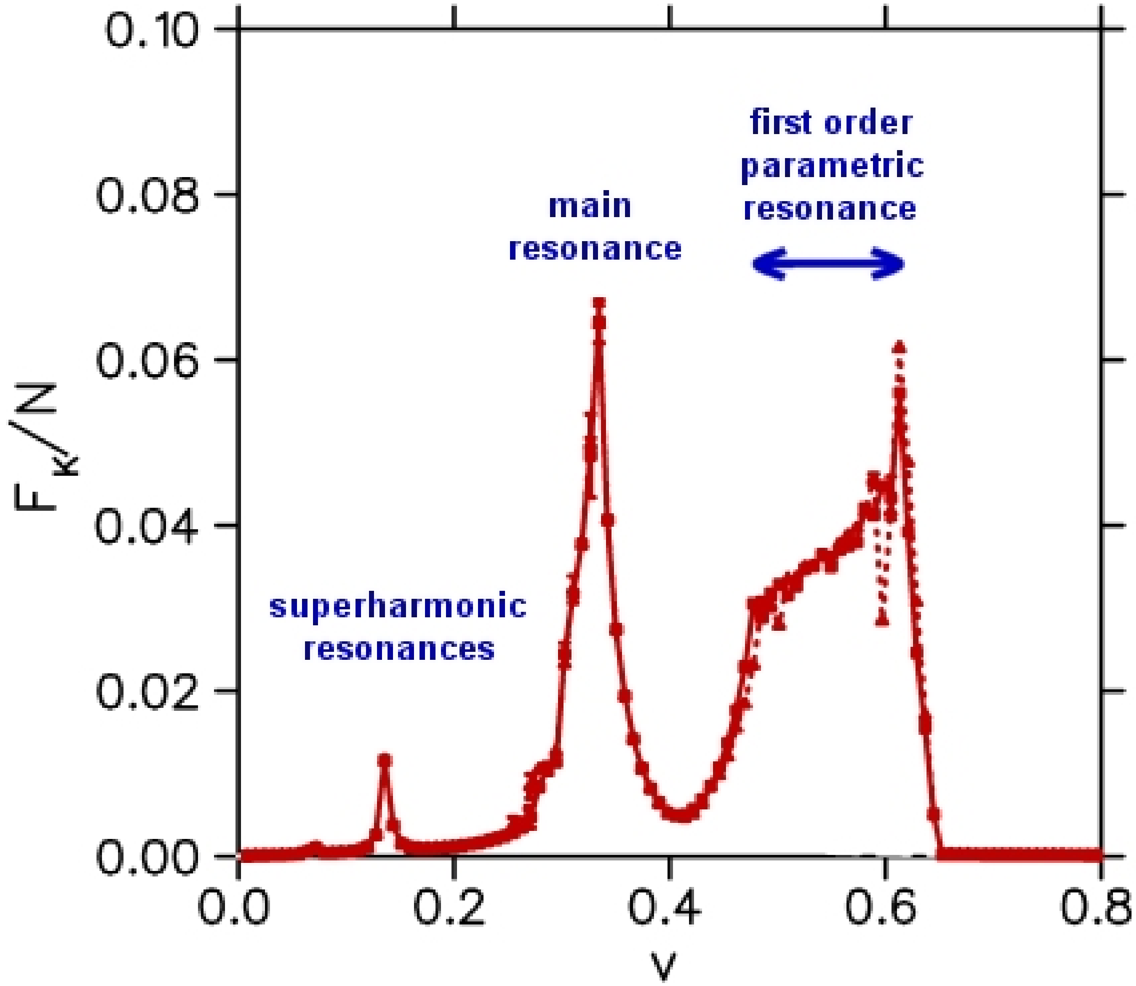


Resonances in the kinetic friction F_K:

FK model

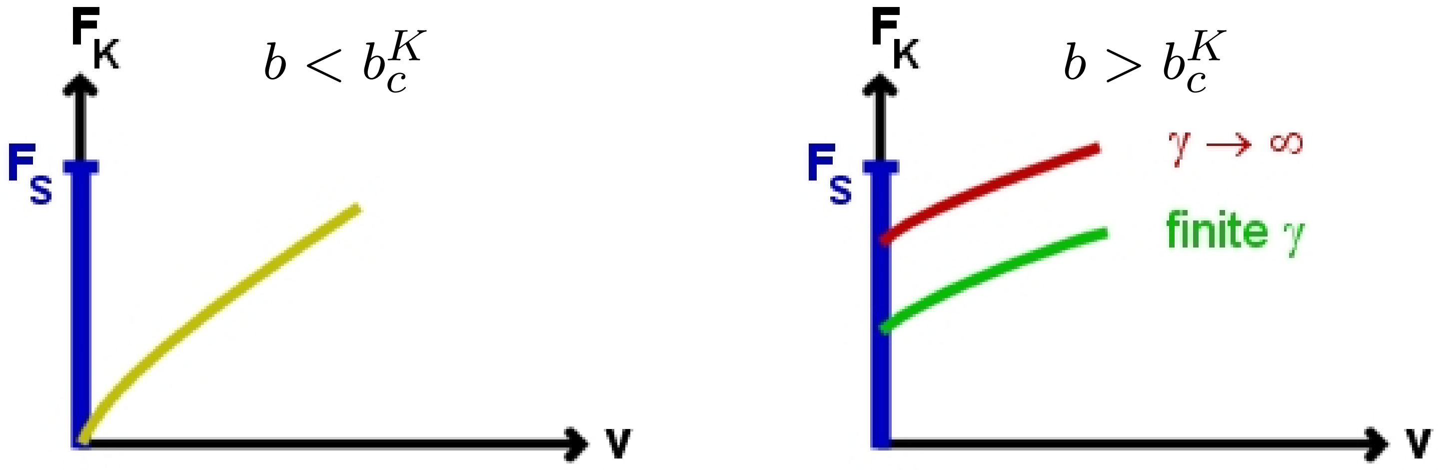


FKT model

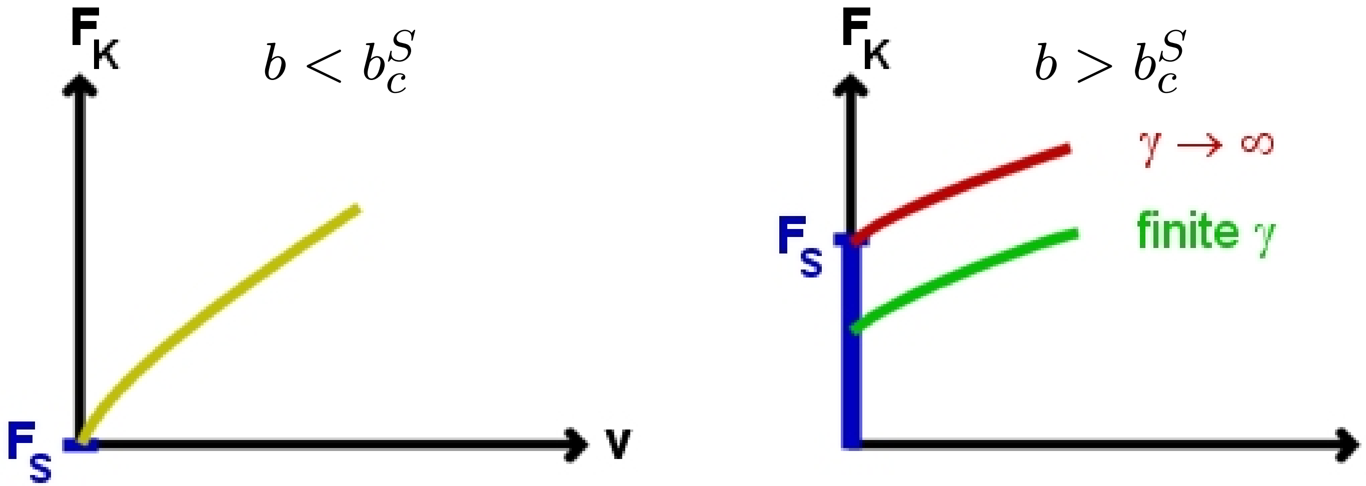


Kinetic friction in the FKT model in the small velocity limit

commensurate case:



incommensurate case:



5. Pinning $\longleftrightarrow$ Sliding

Pinning $\rightarrow$ sliding transition:

- Happens at a saddle-node bifurcation
- Usually starts at a certain particle
- Leads to an avalanche involving neighbouring particles
- **Microslip**: Avalanche stops before sweeping over all particles
- **Pinning $\rightarrow$ sliding transition**: A system-spanning avalanche

Microslip in the FK model

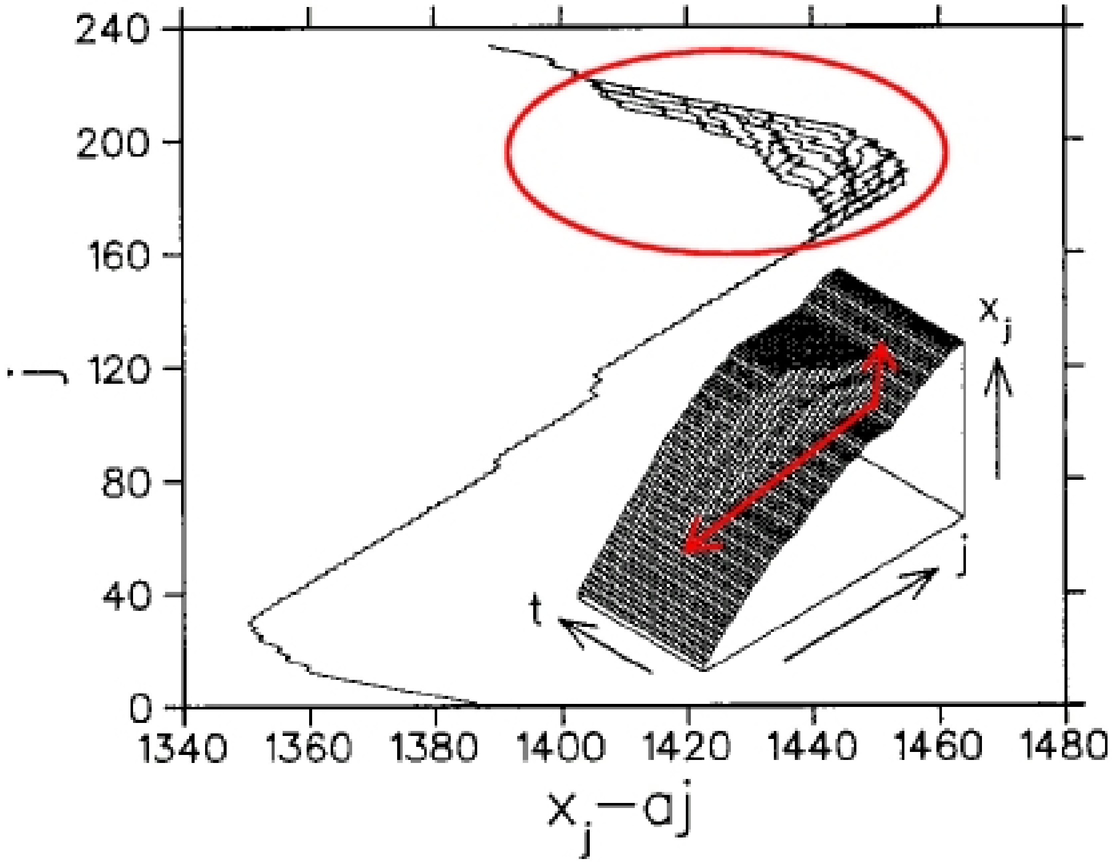
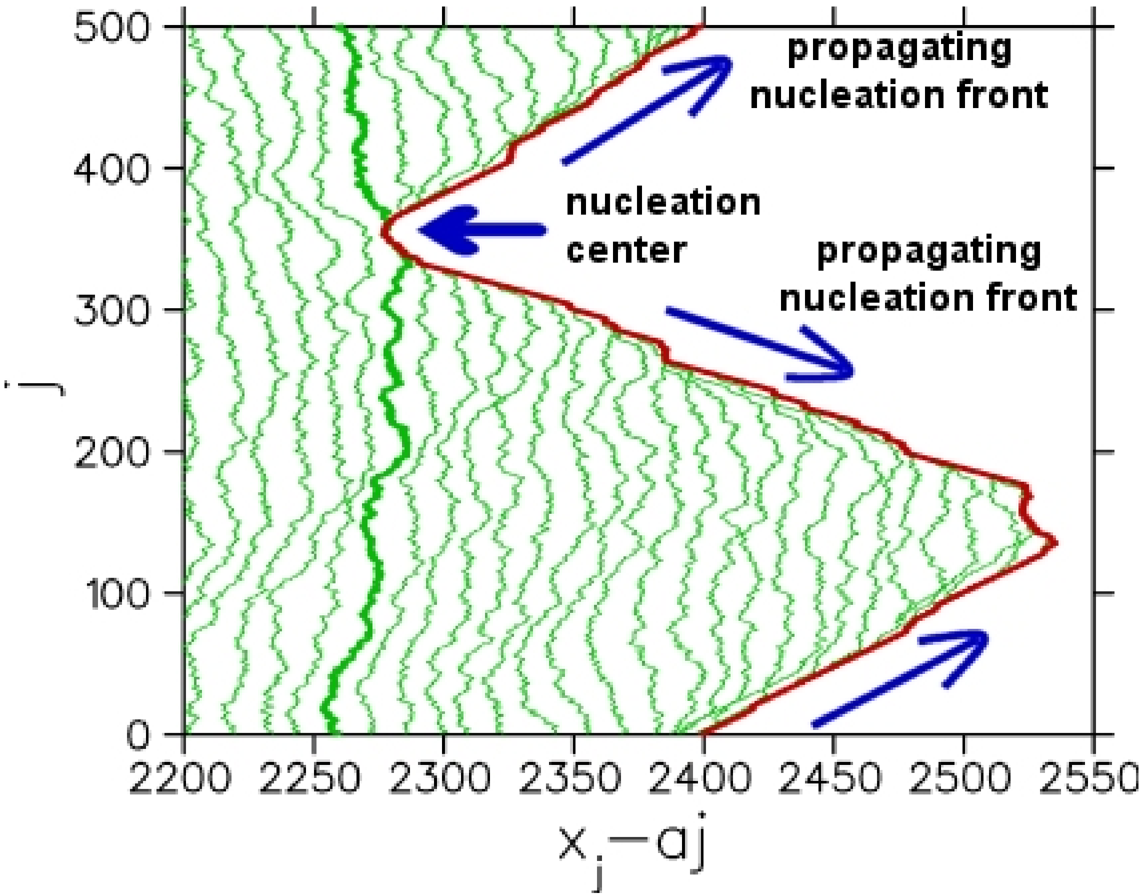


FIG. 13. An example of a microslip. Snapshots at an interval of ten time units are shown. The inset shows x_j for $j \in [160,233]$. The parameters are $N=233$, $M=89$, $b=2$, $\gamma=0.05$, and F from 0.115 54 until 0.115 575, at a rate of 10^{-7}.

Sliding $\rightarrow$ pinning transition in the FK model

Similar to an ordinary first order phase transition:



1. creation of a critical nucleus: nucleation time t_N is Poisson-distributed

$$\langle t_N \rangle = \Delta t_N \sim 1/N$$

2. growth of the nucleus: growth time t_G

$$\langle t_G \rangle \sim N$$

6. Why studying toy models?

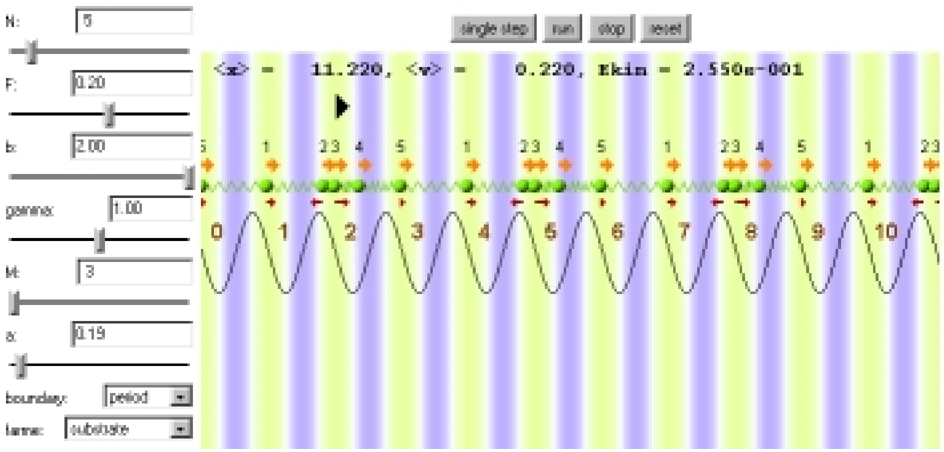
Or in terms of todays question in the Round Table Discussion:
Are abstract models useful for anything?

Yes, because toy models are

- conceptual simple
- laboratories for basic concepts
- analytical treatable, at least sometimes ;-)
- less expensive for simulation (a PC instead of a super computer is enough)

The Friction Lab

http://monet.physik.unibas.ch/~elmer/flab



The FK model as a Java Applet. Other models in the future.